Maple 2018.2 Integration Test Results
on the problems in "5 Inverse trig functions/5.5 Inverse secant"

Test results for the 48 problems in "5.5.1 u (a+b arcsec(c x))^n.txt"

Problem 4: Result more than twice size of optimal antiderivative.

$$\int \frac{a+b\,\mathrm{arcsec}(c x)}{x^3}\, dx$$

Optimal(type 3, 46 leaves, 4 steps):

$$-\frac{b c^2 \,\mathrm{arccsc}(c x)}{4}+\frac{-a-b\,\mathrm{arcsec}(c x)}{2 x^2}+\frac{b c \sqrt{1-\frac{1}{c^2 x^2}}}{4 x}$$

Result(type 3, 117 leaves):

$$-\frac{a}{2 x^2}-\frac{b\,\mathrm{arcsec}(c x)}{2 x^2}-\frac{c b \sqrt{c^2 x^2-1}\,\arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right)}{4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}+\frac{c b}{4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}-\frac{b}{4 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x^3}$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int x^3 \left(a+b\,\mathrm{arcsec}(c x)\right)^2 dx$$

Optimal(type 3, 93 leaves, 5 steps):

$$\frac{b^2 x^2}{12 c^2}+\frac{x^4 \left(a+b\,\mathrm{arcsec}(c x)\right)^2}{4}+\frac{b^2 \ln(x)}{3 c^4}-\frac{b x \left(a+b\,\mathrm{arcsec}(c x)\right) \sqrt{1-\frac{1}{c^2 x^2}}}{3 c^3}-\frac{b x^3 \left(a+b\,\mathrm{arcsec}(c x)\right) \sqrt{1-\frac{1}{c^2 x^2}}}{6 c}$$

Result(type 3, 207 leaves):

$$\frac{x^4 a^2}{4}+\frac{b^2 \,\mathrm{arcsec}(c x)^2 x^4}{4}-\frac{b^2 \,\mathrm{arcsec}(c x) \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x^3}{6 c}+\frac{b^2 x^2}{12 c^2}-\frac{b^2 \,\mathrm{arcsec}(c x)\, x \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}{3 c^3}-\frac{b^2 \ln\left(\frac{1}{c x}\right)}{3 c^4}+\frac{a b x^4 \,\mathrm{arcsec}(c x)}{2}$$
$$-\frac{a b x^3}{6 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{a b x}{6 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}+\frac{a b}{3 c^5 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

Problem 6: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(a+b\,\mathrm{arcsec}(c x)\right)^2 dx$$

Optimal(type 4, 167 leaves, 8 steps):

$$\frac{b^2 x}{3 c^2}+\frac{x^3 (a+b \operatorname{arcsec}(c x))^2}{3}+\frac{2 I b (a+b \operatorname{arcsec}(c x)) \arctan\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)}{3 c^3}-\frac{I b^2 \operatorname{polylog}\left(2,-I\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)\right)}{3 c^3}$$
$$+\frac{I b^2 \operatorname{polylog}\left(2, I\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)\right)}{3 c^3}-\frac{b x^2 (a+b \operatorname{arcsec}(c x)) \sqrt{1-\frac{1}{c^2 x^2}}}{3 c}$$

Result(type 4, 342 leaves):

$$\frac{x^3 a^2}{3}+\frac{x^3 b^2 \operatorname{arcsec}(c x)^2}{3}-\frac{b^2 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}} \operatorname{arcsec}(c x) x^2}{3 c}+\frac{b^2 x}{3 c^2}+\frac{b^2 \operatorname{arcsec}(c x) \ln\left(1+I\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)\right)}{3 c^3}$$
$$-\frac{b^2 \operatorname{arcsec}(c x) \ln\left(1-I\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)\right)}{3 c^3}-\frac{I b^2 \operatorname{dilog}\left(1+I\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)\right)}{3 c^3}+\frac{I b^2 \operatorname{dilog}\left(1-I\left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right)\right)}{3 c^3}$$
$$+\frac{2 x^3 a b \operatorname{arcsec}(c x)}{3}-\frac{a b x^2}{3 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}+\frac{a b}{3 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{a b \sqrt{c^2 x^2-1} \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{3 c^4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

Problem 7: Result more than twice size of optimal antiderivative.

$$\int x (a+b \operatorname{arcsec}(c x))^2 \, dx$$

Optimal(type 3, 52 leaves, 4 steps):

$$\frac{x^2 (a+b \operatorname{arcsec}(c x))^2}{2}+\frac{b^2 \ln(x)}{c^2}-\frac{b x (a+b \operatorname{arcsec}(c x)) \sqrt{1-\frac{1}{c^2 x^2}}}{c}$$

Result(type 3, 133 leaves):

$$\frac{x^2 a^2}{2}+\frac{b^2 x^2 \operatorname{arcsec}(c x)^2}{2}-\frac{b^2 \operatorname{arcsec}(c x)\, x \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}{c}-\frac{b^2 \ln\left(\frac{1}{c x}\right)}{c^2}+a b x^2 \operatorname{arcsec}(c x)-\frac{a b x}{c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}+\frac{a b}{c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arcsec}(c x))^2}{x^5} \, dx$$

Optimal(type 3, 116 leaves, 5 steps):

$$\frac{b^2}{32 x^4}+\frac{3 b^2 c^2}{32 x^2}+\frac{3 a b c^4 \operatorname{arcsec}(c x)}{16}+\frac{3 b^2 c^4 \operatorname{arcsec}(c x)^2}{32}-\frac{(a+b \operatorname{arcsec}(c x))^2}{4 x^4}+\frac{b c (a+b \operatorname{arcsec}(c x)) \sqrt{1-\frac{1}{c^2 x^2}}}{8 x^3}+\frac{3 b c^3 (a+b \operatorname{arcsec}(c x)) \sqrt{1-\frac{1}{c^2 x^2}}}{16 x}$$

Result(type 3, 264 leaves):

$$-\frac{a^2}{4 x^4}-\frac{b^2 \operatorname{arcsec}(c x)^2}{4 x^4}+\frac{3 b^2 c^4 \operatorname{arcsec}(c x)^2}{32}+\frac{3 c^3 b^2 \operatorname{arcsec}(c x) \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}{16 x}+\frac{c b^2 \operatorname{arcsec}(c x) \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}{8 x^3}+\frac{b^2}{32 x^4}+\frac{3 b^2 c^2}{32 x^2}-\frac{a b \operatorname{arcsec}(c x)}{2 x^4}-\frac{3 c^3 a b \sqrt{c^2 x^2-1} \arctan\left(\frac{1}{\sqrt{c^2 x^2-1}}\right)}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}+\frac{3 c^3 a b}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}-\frac{c a b}{16 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x^3}-\frac{a b}{8 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x^5}$$

Problem 11: Result more than twice size of optimal antiderivative.

$$\int \frac{(a+b \operatorname{arcsec}(c x))^3}{x^2}\, dx$$

Optimal(type 3, 76 leaves, 5 steps):

$$\frac{6 b^2 (a+b \operatorname{arcsec}(c x))}{x}-\frac{(a+b \operatorname{arcsec}(c x))^3}{x}-6 b^3 c \sqrt{1-\frac{1}{c^2 x^2}}+3 b c (a+b \operatorname{arcsec}(c x))^2 \sqrt{1-\frac{1}{c^2 x^2}}$$

Result(type 3, 197 leaves):

$$c\left(-\frac{a^3}{c x}+b^3\left(-\frac{\operatorname{arcsec}(c x)^3}{c x}+3 \operatorname{arcsec}(c x)^2 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}-6 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}+\frac{6 \operatorname{arcsec}(c x)}{c x}\right)+3 a b^2\left(-\frac{\operatorname{arcsec}(c x)^2}{c x}+\frac{2}{c x}+2 \operatorname{arcsec}(c x) \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\right)+3 a^2 b\left(-\frac{\operatorname{arcsec}(c x)}{c x}+\frac{c^2 x^2-1}{\sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, c^2 x^2}\right)\right)$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int (e x+d)^3 (a+b \operatorname{arcsec}(c x))\, dx$$

Optimal(type 3, 147 leaves, 11 steps):

$$\frac{b\,d^4\operatorname{arccsc}(c x)}{4 e}+\frac{(e x+d)^4\,(a+b\operatorname{arcsec}(c x))}{4 e}-\frac{b\,d\,(2 c^2 d^2+e^2)\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2 x^2}}\right)}{2 c^3}-\frac{b\,e\,(9 c^2 d^2+e^2)\,x\sqrt{1-\frac{1}{c^2 x^2}}}{6 c^3}-\frac{b\,d\,e^2 x^2\sqrt{1-\frac{1}{c^2 x^2}}}{2 c}$$
$$-\frac{b\,e^3 x^3\sqrt{1-\frac{1}{c^2 x^2}}}{12 c}$$

Result(type 3, 485 leaves):

$$\frac{a e^3 x^4}{4}+a e^2 x^3 d+\frac{3 a e x^2 d^2}{2}+a x d^3+\frac{a d^4}{4 e}+\frac{b e^3\operatorname{arcsec}(c x)\,x^4}{4}+b e^2\operatorname{arcsec}(c x)\,x^3 d+\frac{3 b e\operatorname{arcsec}(c x)\,x^2 d^2}{2}+b\operatorname{arcsec}(c x)\,x d^3+\frac{b\operatorname{arcsec}(c x)\,d^4}{4 e}$$
$$+\frac{b\sqrt{c^2x^2-1}\,d^4\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{4 c e\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b\sqrt{c^2x^2-1}\,d^3\ln\left(c x+\sqrt{c^2x^2-1}\right)}{c^2\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b e^3 x^3}{12 c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b e^3 x}{12 c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b e^2 d x^2}{2 c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$
$$+\frac{b e^2 d}{2 c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3 b e x d^2}{2 c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{3 b e d^2}{2 c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}-\frac{b e^2\sqrt{c^2x^2-1}\,d\ln\left(c x+\sqrt{c^2x^2-1}\right)}{2 c^4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{b e^3}{6 c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$$

Problem 23: Result more than twice size of optimal antiderivative.

$$\int x^2\,(e x^2+d)\,(a+b\operatorname{arcsec}(c x))\,\mathrm{d}x$$

Optimal(type 3, 139 leaves, 6 steps):

$$\frac{d x^3\,(a+b\operatorname{arcsec}(c x))}{3}+\frac{e x^5\,(a+b\operatorname{arcsec}(c x))}{5}-\frac{b\,(20 c^2 d+9 e)\,x\operatorname{arctanh}\left(\frac{c x}{\sqrt{c^2x^2-1}}\right)}{120 c^4\sqrt{c^2x^2}}-\frac{b\,(20 c^2 d+9 e)\,x^2\sqrt{c^2x^2-1}}{120 c^3\sqrt{c^2x^2}}-\frac{b e x^4\sqrt{c^2x^2-1}}{20 c\sqrt{c^2x^2}}$$

Result(type 3, 281 leaves):

$$\frac{a e x^5}{5}+\frac{a x^3 d}{3}+\frac{b\operatorname{arcsec}(c x)\,e x^5}{5}+\frac{b\operatorname{arcsec}(c x)\,x^3 d}{3}-\frac{b x^4 e}{20 c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b x^2 e}{40 c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{b d x^2}{6 c\sqrt{\frac{c^2x^2-1}{c^2x^2}}}+\frac{b d}{6 c^3\sqrt{\frac{c^2x^2-1}{c^2x^2}}}$$
$$-\frac{b\sqrt{c^2x^2-1}\,d\ln\left(c x+\sqrt{c^2x^2-1}\right)}{6 c^4\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}+\frac{3 b e}{40 c^5\sqrt{\frac{c^2x^2-1}{c^2x^2}}}-\frac{3 b\sqrt{c^2x^2-1}\,e\ln\left(c x+\sqrt{c^2x^2-1}\right)}{40 c^6\sqrt{\frac{c^2x^2-1}{c^2x^2}}\,x}$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2 \left(e x^2+d\right)^2 \left(a+b \operatorname{arcsec}(c x)\right) \, dx$$

Optimal(type 3, 222 leaves, 7 steps):

$$\frac{d^2 x^3 \left(a+b \operatorname{arcsec}(c x)\right)}{3}+\frac{2 d e x^5 \left(a+b \operatorname{arcsec}(c x)\right)}{5}+\frac{e^2 x^7 \left(a+b \operatorname{arcsec}(c x)\right)}{7}-\frac{b \left(280 c^4 d^2+252 c^2 d e+75 e^2\right) x \operatorname{arctanh}\left(\frac{c x}{\sqrt{c^2 x^2-1}}\right)}{1680 c^6 \sqrt{c^2 x^2}}$$
$$-\frac{b \left(280 c^4 d^2+252 c^2 d e+75 e^2\right) x^2 \sqrt{c^2 x^2-1}}{1680 c^5 \sqrt{c^2 x^2}}-\frac{b e \left(84 c^2 d+25 e\right) x^4 \sqrt{c^2 x^2-1}}{840 c^3 \sqrt{c^2 x^2}}-\frac{b e^2 x^6 \sqrt{c^2 x^2-1}}{42 c \sqrt{c^2 x^2}}$$

Result(type 3, 493 leaves):

$$\frac{a e^2 x^7}{7}+\frac{2 a d e x^5}{5}+\frac{a d^2 x^3}{3}+\frac{b \operatorname{arcsec}(c x) e^2 x^7}{7}+\frac{2 b \operatorname{arcsec}(c x) d e x^5}{5}+\frac{b \operatorname{arcsec}(c x) d^2 x^3}{3}-\frac{b x^6 e^2}{42 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{b x^4 e^2}{168 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}$$
$$-\frac{b x^4 d e}{10 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{b x^2 d e}{20 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{b d^2 x^2}{6 c \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}+\frac{b d^2}{6 c^3 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{b \sqrt{c^2 x^2-1}\, d^2 \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{6 c^4 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}-\frac{5 b x^2 e^2}{336 c^5 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}$$
$$+\frac{3 b d e}{20 c^5 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{3 b \sqrt{c^2 x^2-1}\, d e \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{20 c^6 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}+\frac{5 b e^2}{112 c^7 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}}-\frac{5 b \sqrt{c^2 x^2-1}\, e^2 \ln\left(c x+\sqrt{c^2 x^2-1}\right)}{112 c^8 \sqrt{\frac{c^2 x^2-1}{c^2 x^2}}\, x}$$

Problem 26: Result is not expressed in closed-form.

$$\int \frac{x^2 \left(a+b \operatorname{arcsec}(c x)\right)}{e x^2+d} \, dx$$

Optimal(type 4, 548 leaves, 25 steps):

$$\frac{x \left(a+b \operatorname{arcsec}(c x)\right)}{e}-\frac{b \operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2 x^2}}\right)}{c e}+\frac{\left(a+b \operatorname{arcsec}(c x)\right) \ln\left(1-\frac{c \left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right) \sqrt{-d}}{2 e^{3/2}}$$
$$-\frac{\left(a+b \operatorname{arcsec}(c x)\right) \ln\left(1+\frac{c \left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right) \sqrt{-d}}{2 e^{3/2}}+\frac{\left(a+b \operatorname{arcsec}(c x)\right) \ln\left(1-\frac{c \left(\frac{1}{c x}+I \sqrt{1-\frac{1}{c^2 x^2}}\right) \sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right) \sqrt{-d}}{2 e^{3/2}}$$

$$-\frac{(a+b\,\mathrm{arcsec}(c x))\ln\left(1+\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{2 e^{3/2}}+\frac{I b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{-d}}{2 e^{3/2}}$$

$$-\frac{I b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)\sqrt{-d}}{2 e^{3/2}}+\frac{I b\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{2 e^{3/2}}$$

$$-\frac{I b\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)\sqrt{-d}}{2 e^{3/2}}$$

Result(type 7, 373 leaves):

$$\frac{a x}{e}-\frac{a d\arctan\left(\frac{x e}{\sqrt{d e}}\right)}{e\sqrt{d e}}+\frac{b\,\mathrm{arcsec}(c x)\,x}{e}+\frac{1}{8 e^2}\Bigg(I c b d\Bigg(\sum_{\_R1=RootOf(c^2 d\_Z^4+(2c^2 d+4e)\_Z^2+c^2 d)}\frac{(\_R1^2 c^2 d+c^2 d+4e)\left(I\,\mathrm{arcsec}(c x)\ln\left(\frac{\_R1-\frac{1}{c x}-I\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{c x}-I\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)\right)}{\_R1\,(\_R1^2 c^2 d+c^2 d+2e)}\Bigg)\Bigg)$$

$$+\frac{2 I b\arctan\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)}{c e}-\frac{1}{8 e^2}\Bigg(I c b d\Bigg($$

$$\sum_{\_R1=RootOf(c^2 d\_Z^4+(2c^2 d+4e)\_Z^2+c^2 d)}$$

$$\frac{\left(\_R1^2 c^2 d + 4\,\_R1^2 e + c^2 d\right)\left(\mathrm{I}\,\mathrm{arcsec}(c x)\ln\left(\frac{\_R1 - \frac{1}{c x} - \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right) + \mathrm{dilog}\left(\frac{\_R1 - \frac{1}{c x} - \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right)\right)}{\_R1\left(\_R1^2 c^2 d + c^2 d + 2 e\right)}\Bigg)\Bigg)$$

Problem 27: Result is not expressed in closed-form.

$$\int \frac{x\,(a + b\,\mathrm{arcsec}(c x))}{e x^2 + d}\,dx$$

Optimal(type 4, 550 leaves, 26 steps):

$$-\frac{(a + b\,\mathrm{arcsec}(c x))\ln\left(1 + \left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)^2\right)}{e} + \frac{(a + b\,\mathrm{arcsec}(c x))\ln\left(1 - \frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2 e}$$

$$+\frac{(a + b\,\mathrm{arcsec}(c x))\ln\left(1 + \frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2 e} + \frac{(a + b\,\mathrm{arcsec}(c x))\ln\left(1 - \frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{2 e}$$

$$+\frac{(a + b\,\mathrm{arcsec}(c x))\ln\left(1 + \frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{2 e} + \frac{\mathrm{I}\,b\,\mathrm{polylog}\left(2, -\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)^2\right)}{2 e}$$

$$-\frac{\mathrm{I}\,b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2 e} - \frac{\mathrm{I}\,b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{2 e}$$

$$-\frac{\mathrm{I}\,b\,\mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{2 e} - \frac{\mathrm{I}\,b\,\mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c x} + \mathrm{I}\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{2 e}$$

Result(type 7, 452 leaves):

$$\frac{a\ln\left(c^2 e x^2 + c^2 d\right)}{2 e}$$

$$-\frac{1}{4e}\left(\mathrm{I}\, c^2 b\, d\left(\sum_{\_R1=RootOf(c^2 d\,\_Z^4+(2c^2 d+4e)\,\_Z^2+c^2 d)}\frac{(\_R1^2+1)\left(\mathrm{I}\operatorname{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{cx}-\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)\right)}{\_R1^2c^2d+c^2d+2e}\right)\right)$$

$$-\frac{b\operatorname{arcsec}(cx)\ln\left(1+\mathrm{I}\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{b\operatorname{arcsec}(cx)\ln\left(1-\mathrm{I}\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}+\frac{\mathrm{I}\, b\operatorname{dilog}\left(1+\mathrm{I}\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}$$

$$+\frac{\mathrm{I}\, b\operatorname{dilog}\left(1-\mathrm{I}\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\right)}{e}-\frac{1}{4e}\left(\mathrm{I}\, b\left(\sum_{\_R1=RootOf(c^2 d\,\_Z^4+(2c^2 d+4e)\,\_Z^2+c^2 d)}\frac{(\_R1^2c^2d+c^2d+4e)\left(\mathrm{I}\operatorname{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{cx}-\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)\right)}{\_R1^2c^2d+c^2d+2e}\right)\right)$$

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Problem 28: Result is not expressed in closed-form.
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$$\int\frac{x^4\,(a+b\operatorname{arcsec}(cx))}{(ex^2+d)^2}\,dx$$

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Optimal(type 4, 742 leaves, 51 steps):
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$$\frac{x\,(a+b\operatorname{arcsec}(cx))}{e^2}-\frac{b\operatorname{arctanh}\left(\sqrt{1-\frac{1}{c^2x^2}}\right)}{c\,e^2}+\frac{3\,(a+b\operatorname{arcsec}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3\left(a+b\,\mathrm{arcsec}(cx)\right)\ln\left(1+\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}+\frac{3\left(a+b\,\mathrm{arcsec}(cx)\right)\ln\left(1-\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3\left(a+b\,\mathrm{arcsec}(cx)\right)\ln\left(1+\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}+\frac{3Ib\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3Ib\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}+\frac{3Ib\,\mathrm{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}$$

$$-\frac{3Ib\,\mathrm{polylog}\left(2,\frac{c\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)\sqrt{-d}}{4e^{5/2}}-\frac{d\left(a+b\,\mathrm{arcsec}(cx)\right)}{4e^2\left(-\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}+\frac{d\left(a+b\,\mathrm{arcsec}(cx)\right)}{4e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

$$-\frac{b\,\mathrm{arctanh}\left(\frac{c^2d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)\sqrt{d}}{4e^2\sqrt{c^2d+e}}-\frac{b\,\mathrm{arctanh}\left(\frac{c^2d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2d+e}\sqrt{1-\frac{1}{c^2x^2}}}\right)\sqrt{d}}{4e^2\sqrt{c^2d+e}}$$

Result(type 7, 1886 leaves):

$$\frac{ax}{e^2}+\frac{c^2adx}{2e^2\left(c^2ex^2+c^2d\right)}-\frac{3ad\arctan\left(\frac{xe}{\sqrt{de}}\right)}{2e^2\sqrt{de}}+\frac{c^2bx^3\,\mathrm{arcsec}(cx)}{\left(c^2ex^2+c^2d\right)e}+\frac{3c^2b\,\mathrm{arcsec}(cx)\,dx}{2e^2\left(c^2ex^2+c^2d\right)}$$

$$+\frac{Ib\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)d}\,\arctan\left(\frac{\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)cd}{\sqrt{\left(c^2d+2\sqrt{e\left(c^2d+e\right)}+2e\right)d}}\right)\sqrt{e\left(c^2d+e\right)}}{c^4e\left(c^2d+e\right)d^2}$$

$$-\frac{I\,b\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}\;\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)c\,d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}}\right)\sqrt{e\,(c^2 d+e)}}{c^4 e^2 d^2}+\frac{2\,I\,b\arctan\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)}{c\,e^2}$$

$$+\frac{I\,b\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}\;\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)c\,d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}}\right)}{2\,c^2 e^2 d}$$

$$-\frac{I\,b\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}\;\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)c\,d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}}\right)}{c^2 e\,(c^2 d+e)\,d}$$

$$-\frac{I\,b\sqrt{-\left(c^2 d-2\sqrt{e\,(c^2 d+e)}+2e\right)d}\;\operatorname{arctanh}\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)c\,d}{\sqrt{\left(-c^2 d+2\sqrt{e\,(c^2 d+e)}-2e\right)d}}\right)\sqrt{e\,(c^2 d+e)}}{c^4 e\,(c^2 d+e)\,d^2}+\frac{1}{16 e^3}\Bigg(3\,I\,c\,b\,d\Bigg($$

$$\sum_{\_R1=RootOf\left(c^2 d\,\_Z^4+(2c^2 d+4e)\,\_Z^2+c^2 d\right)}\frac{\left(\_R1^2 c^2 d+c^2 d+4e\right)\left(I\operatorname{arcsec}(c x)\ln\left(\frac{\_R1-\frac{1}{c x}-I\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{c x}-I\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)\right)}{\_R1\left(\_R1^2 c^2 d+c^2 d+2e\right)}\Bigg)\Bigg)$$

$$+\frac{I\,b\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}\;\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)c\,d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2e\right)d}}\right)}{c^4 e\,d^2}$$

$$+\frac{I\,b\sqrt{-\left(c^2 d-2\sqrt{e\,(c^2 d+e)}+2e\right)d}\;\operatorname{arctanh}\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)c\,d}{\sqrt{\left(-c^2 d+2\sqrt{e\,(c^2 d+e)}-2e\right)d}}\right)}{2\,c^2 e^2 d}$$

$$- \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right)}{c^2 e \left(c^2 d + e\right) d}$$

$$- \frac{I b \sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \arctan\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}}\right)}{c^4 \left(c^2 d + e\right) d^2}$$

$$+ \frac{I b \sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \arctan\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{2 c^2 e^2 \left(c^2 d + e\right) d}$$

$$+ \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right)}{c^4 e d^2}$$

$$- \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{2 c^2 e^2 \left(c^2 d + e\right) d}$$

$$- \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right)}{c^4 \left(c^2 d + e\right) d^2}$$

$$+ \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\, \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{c^4 e^2 d^2} - \frac{1}{16 e^3} \left(3 I c b d \left(\right.\right.$$

$$\sum_{\_R1=RootOf(c^2 d\_Z^4+(2c^2 d+4e)\_Z^2+c^2 d)} \frac{(\_R1^2 c^2 d+4\_R1^2 e+c^2 d)\left(\mathrm{I}\operatorname{arcsec}(cx)\ln\left(\frac{\_R1-\frac{1}{cx}-\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{cx}-\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)\right)}{\_R1\,(\_R1^2 c^2 d+c^2 d+2e)}\Bigg)\Bigg)$$

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Problem 29: Result is not expressed in closed-form.
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$$\int \frac{x^2\,(a+b\operatorname{arcsec}(cx))}{(ex^2+d)^2}\,dx$$

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Optimal(type 4, 708 leaves, 27 steps):
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$$\frac{(a+b\operatorname{arcsec}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}-\frac{(a+b\operatorname{arcsec}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}$$

$$+\frac{(a+b\operatorname{arcsec}(cx))\ln\left(1-\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}-\frac{(a+b\operatorname{arcsec}(cx))\ln\left(1+\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}$$

$$+\frac{\mathrm{I}\,b\operatorname{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}-\frac{\mathrm{I}\,b\operatorname{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}$$

$$+\frac{\mathrm{I}\,b\operatorname{polylog}\left(2,-\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}-\frac{\mathrm{I}\,b\operatorname{polylog}\left(2,\frac{c\left(\frac{1}{cx}+\mathrm{I}\sqrt{1-\frac{1}{c^2x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2d+e}}\right)}{4e^{3/2}\sqrt{-d}}+\frac{a+b\operatorname{arcsec}(cx)}{4e\left(-\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

$$+\frac{-a-b\,\mathrm{arcsec}(c x)}{4 e\left(\frac{d}{x}+\sqrt{-d}\,\sqrt{e}\right)}+\frac{b\,\mathrm{arctanh}\left(\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2 x^2}}}\right)}{4 e\sqrt{d}\,\sqrt{c^2 d+e}}+\frac{b\,\mathrm{arctanh}\left(\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d+e}\,\sqrt{1-\frac{1}{c^2 x^2}}}\right)}{4 e\sqrt{d}\,\sqrt{c^2 d+e}}$$

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Result(type 7, 1755 leaves):
```

$$-\frac{c^2 a x}{2 e\left(c^2 e x^2+c^2 d\right)}+\frac{a\arctan\left(\frac{x e}{\sqrt{d e}}\right)}{2 e\sqrt{d e}}-\frac{c^2 b\,\mathrm{arcsec}(c x)\,x}{2 e\left(c^2 e x^2+c^2 d\right)}$$

$$+\frac{\mathrm{I}\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+\mathrm{I}\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right)\sqrt{e\left(c^2 d+e\right)}}{c^4 e d^3}$$

$$-\frac{\mathrm{I}\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+\mathrm{I}\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right)}{2 c^2 e d^2}$$

$$-\frac{\mathrm{I}\, c b\left(\sum_{\_R1=RootOf\left(c^2 d\,\_Z^4+\left(2 c^2 d+4 e\right)\_Z^2+c^2 d\right)}\frac{\mathrm{I}\,\mathrm{arcsec}(c x)\ln\left(\frac{\_R1-\frac{1}{c x}-\mathrm{I}\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-\frac{1}{c x}-\mathrm{I}\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)}{\_R1\left(\_R1^2 c^2 d+c^2 d+2 e\right)}\right)}{4 e}$$

$$-\frac{\mathrm{I}\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+\mathrm{I}\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right)\sqrt{e\left(c^2 d+e\right)}}{2 c^2 e\left(c^2 d+e\right) d^2}$$

$$+\frac{\mathrm{I}\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+\mathrm{I}\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right) e}{c^4\left(c^2 d+e\right) d^3}$$

$$- \frac{I b \sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \arctan\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{c^4 \left(c^2 d + e\right) d^3}$$

$$- \frac{I b \sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \arctan\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}}\right)}{c^4 d^3}$$

$$- \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{c^4 e d^3}$$

$$+ \frac{I b \sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \arctan\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d + 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}}\right)}{c^2 \left(c^2 d + e\right) d^2}$$

$$+ \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{c^4 \left(c^2 d + e\right) d^3}$$

$$+ \frac{I c b \left(\sum_{\_R1 = RootOf\left(c^2 d\ \_Z^4 + \left(2 c^2 d + 4 e\right) \_Z^2 + c^2 d\right)} \frac{\_R1 \left(I \operatorname{arcsec}(c x) \ln\left(\frac{\_R1 - \frac{1}{c x} - I\sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right) + \operatorname{dilog}\left(\frac{\_R1 - \frac{1}{c x} - I\sqrt{1 - \frac{1}{c^2 x^2}}}{\_R1}\right)\right)}{\_R1^2 c^2 d + c^2 d + 2 e}\right)}{4 e}$$

$$+ \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right) \sqrt{e\left(c^2 d + e\right)}}{2 c^2 e \left(c^2 d + e\right) d^2}$$

$$- \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right)}{2 c^2 e d^2}$$

$$- \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right)}{c^4 d^3}$$

$$+ \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right) e}{c^4 \left(c^2 d + e\right) d^3}$$

$$+ \frac{I b \sqrt{-\left(c^2 d - 2\sqrt{e\left(c^2 d + e\right)} + 2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d + 2\sqrt{e\left(c^2 d + e\right)} - 2 e\right) d}}\right)}{c^2 \left(c^2 d + e\right) d^2}$$

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Problem 30: Result is not expressed in closed-form.
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$$\int \frac{a + b \operatorname{arcsec}(c x)}{\left(e x^2 + d\right)^2}\, dx$$

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Optimal(type 4, 702 leaves, 47 steps):
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$$- \frac{\left(a + b \operatorname{arcsec}(c x)\right) \ln\left(1 - \frac{c\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{4 (-d)^{3/2}\sqrt{e}} + \frac{\left(a + b \operatorname{arcsec}(c x)\right) \ln\left(1 + \frac{c\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} - \sqrt{c^2 d + e}}\right)}{4 (-d)^{3/2}\sqrt{e}}$$

$$- \frac{\left(a + b \operatorname{arcsec}(c x)\right) \ln\left(1 - \frac{c\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{4 (-d)^{3/2}\sqrt{e}} + \frac{\left(a + b \operatorname{arcsec}(c x)\right) \ln\left(1 + \frac{c\left(\frac{1}{c x} + I\sqrt{1 - \frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e} + \sqrt{c^2 d + e}}\right)}{4 (-d)^{3/2}\sqrt{e}}$$

$$-\frac{I\, b\, \mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{4\,(-d)^{3/2}\sqrt{e}}+\frac{I\, b\, \mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}-\sqrt{c^2 d+e}}\right)}{4\,(-d)^{3/2}\sqrt{e}}$$

$$-\frac{I\, b\, \mathrm{polylog}\left(2, -\frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{4\,(-d)^{3/2}\sqrt{e}}+\frac{I\, b\, \mathrm{polylog}\left(2, \frac{c\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right)\sqrt{-d}}{\sqrt{e}+\sqrt{c^2 d+e}}\right)}{4\,(-d)^{3/2}\sqrt{e}}+\frac{-a-b\,\mathrm{arcsec}(c x)}{4 d\left(-\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}$$

$$+\frac{a+b\,\mathrm{arcsec}(c x)}{4 d\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)}-\frac{b\,\mathrm{arctanh}\left(\frac{c^2 d-\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d+e}\sqrt{1-\frac{1}{c^2 x^2}}}\right)}{4 d^{3/2}\sqrt{c^2 d+e}}-\frac{b\,\mathrm{arctanh}\left(\frac{c^2 d+\frac{\sqrt{-d}\sqrt{e}}{x}}{c\sqrt{d}\sqrt{c^2 d+e}\sqrt{1-\frac{1}{c^2 x^2}}}\right)}{4 d^{3/2}\sqrt{c^2 d+e}}$$

Result(type 7, 1747 leaves):

$$\frac{c^2 a x}{2 d\left(c^2 e x^2+c^2 d\right)}+\frac{a\arctan\left(\frac{x e}{\sqrt{d e}}\right)}{2 d\sqrt{d e}}+\frac{c^2 b\,\mathrm{arcsec}(c x)\, x}{2 d\left(c^2 e x^2+c^2 d\right)}+\frac{I\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right) e}{c^4 d^4}$$

$$-\frac{I\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right) e^2}{c^4 d^4\left(c^2 d+e\right)}$$

$$-\frac{I\, b\sqrt{-\left(c^2 d-2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\mathrm{arctanh}\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(-c^2 d+2\sqrt{e\left(c^2 d+e\right)}-2 e\right) d}}\right) e}{c^2\left(c^2 d+e\right) d^3}$$

$$-\frac{I\, b\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}\,\arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c d}{\sqrt{\left(c^2 d+2\sqrt{e\left(c^2 d+e\right)}+2 e\right) d}}\right)\sqrt{e\left(c^2 d+e\right)}}{c^4 d^4}$$

$$+\frac{I\, b \sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2 e\right) d}\ \arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c\, d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2 e\right) d}}\right)\sqrt{e\,(c^2 d+e)}\ e}{c^4 d^4 (c^2 d+e)}$$

$$+\frac{I\, b \sqrt{-\left(c^2 d-2\sqrt{e\,(c^2 d+e)}+2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c\, d}{\sqrt{\left(-c^2 d+2\sqrt{e\,(c^2 d+e)}-2 e\right) d}}\right) e}{c^4 d^4}$$

$$+\frac{I\, b \sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2 e\right) d}\ \arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c\, d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2 e\right) d}}\right)}{2 c^2 d^3}$$

$$-\frac{I\, b \sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2 e\right) d}\ \arctan\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c\, d}{\sqrt{\left(c^2 d+2\sqrt{e\,(c^2 d+e)}+2 e\right) d}}\right) e}{c^2 (c^2 d+e)\, d^3}$$

$$-\frac{I\, b \sqrt{-\left(c^2 d-2\sqrt{e\,(c^2 d+e)}+2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c\, d}{\sqrt{\left(-c^2 d+2\sqrt{e\,(c^2 d+e)}-2 e\right) d}}\right)\sqrt{e\,(c^2 d+e)}\ e}{c^4 d^4 (c^2 d+e)}$$

$$+\frac{I\, c\, b \left(\sum_{\_R1=RootOf(c^2 d\, \_Z^4+(2 c^2 d+4 e)\, \_Z^2+c^2 d)} \frac{\_R1 \left(I \operatorname{arcsec}(c x) \ln\left(\frac{\_R1-\frac{1}{c x}-I\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{c x}-I\sqrt{1-\frac{1}{c^2 x^2}}}{\_R1}\right)\right)}{\_R1^2 c^2 d+c^2 d+2 e}\right)}{4 d}$$

$$-\frac{I\, b \sqrt{-\left(c^2 d-2\sqrt{e\,(c^2 d+e)}+2 e\right) d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{c x}+I\sqrt{1-\frac{1}{c^2 x^2}}\right) c\, d}{\sqrt{\left(-c^2 d+2\sqrt{e\,(c^2 d+e)}-2 e\right) d}}\right)\sqrt{e\,(c^2 d+e)}}{2 c^2 (c^2 d+e)\, d^3}$$

$$-\frac{I c b \left(\sum_{\_R1=RootOf(c^2 d\_Z^4+(2c^2 d+4e)\_Z^2+c^2 d)} \frac{I \operatorname{arcsec}(cx) \ln\left(\frac{\_R1-\frac{1}{cx}-I\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-\frac{1}{cx}-I\sqrt{1-\frac{1}{c^2x^2}}}{\_R1}\right)}{\_R1\left(\_R1^2 c^2 d+c^2 d+2e\right)}\right)}{4d}$$

$$+\frac{I b\sqrt{-\left(c^2 d-2\sqrt{e(c^2 d+e)}+2e\right)d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)cd}{\sqrt{\left(-c^2 d+2\sqrt{e(c^2 d+e)}-2e\right)d}}\right)\sqrt{e(c^2 d+e)}}{c^4 d^4}$$

$$-\frac{I b\sqrt{-\left(c^2 d-2\sqrt{e(c^2 d+e)}+2e\right)d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)cd}{\sqrt{\left(-c^2 d+2\sqrt{e(c^2 d+e)}-2e\right)d}}\right)e^2}{c^4 d^4 (c^2 d+e)}$$

$$+\frac{I b\sqrt{-\left(c^2 d-2\sqrt{e(c^2 d+e)}+2e\right)d}\ \operatorname{arctanh}\left(\frac{\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)cd}{\sqrt{\left(-c^2 d+2\sqrt{e(c^2 d+e)}-2e\right)d}}\right)}{2c^2 d^3}$$

$$+\frac{I b\sqrt{\left(c^2 d+2\sqrt{e(c^2 d+e)}+2e\right)d}\ \arctan\left(\frac{\left(\frac{1}{cx}+I\sqrt{1-\frac{1}{c^2x^2}}\right)cd}{\sqrt{\left(c^2 d+2\sqrt{e(c^2 d+e)}+2e\right)d}}\right)\sqrt{e(c^2 d+e)}}{2c^2(c^2 d+e)d^3}$$

Problem 31: Unable to integrate problem.

$$\int x\left(a+b\operatorname{arcsec}(cx)\right)\sqrt{ex^2+d}\,dx$$

Optimal(type 3, 159 leaves, 9 steps):

$$\frac{(ex^2+d)^{3/2}\left(a+b\operatorname{arcsec}(cx)\right)}{3e}+\frac{b c d^{3/2} x \arctan\left(\frac{\sqrt{ex^2+d}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{3e\sqrt{c^2x^2}}-\frac{b(3c^2 d+e)x \operatorname{arctanh}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{ex^2+d}}\right)}{6c^2\sqrt{e}\sqrt{c^2x^2}}-\frac{b x\sqrt{c^2x^2-1}\sqrt{ex^2+d}}{6c\sqrt{c^2x^2}}$$

Result(type 8, 21 leaves):

$$\int x\left(a+b\operatorname{arcsec}(cx)\right)\sqrt{ex^2+d}\,dx$$

Problem 34: Unable to integrate problem.

$$\int \frac{(a+b\,\mathrm{arcsec}(c x))\sqrt{e x^2+d}}{x^4}\,dx$$

Optimal(type 4, 286 leaves, 11 steps):

$$-\frac{(e x^2+d)^{3/2}(a+b\,\mathrm{arcsec}(c x))}{3 d x^3}+\frac{2 b c (c^2 d+2 e)\sqrt{c^2 x^2-1}\,\sqrt{e x^2+d}}{9 d \sqrt{c^2 x^2}}+\frac{b c\sqrt{c^2 x^2-1}\,\sqrt{e x^2+d}}{9 x^2\sqrt{c^2 x^2}}$$
$$-\frac{2 b c^2 (c^2 d+2 e)\, x\, \mathrm{EllipticE}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right)\sqrt{-c^2 x^2+1}\,\sqrt{e x^2+d}}{9 d\sqrt{c^2 x^2}\,\sqrt{c^2 x^2-1}\,\sqrt{1+\frac{e x^2}{d}}}$$
$$+\frac{b (c^2 d+e)(2 c^2 d+3 e)\, x\, \mathrm{EllipticF}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right)\sqrt{-c^2 x^2+1}\,\sqrt{1+\frac{e x^2}{d}}}{9 d\sqrt{c^2 x^2}\,\sqrt{c^2 x^2-1}\,\sqrt{e x^2+d}}$$

Result(type 8, 23 leaves):

$$\int \frac{(a+b\,\mathrm{arcsec}(c x))\sqrt{e x^2+d}}{x^4}\,dx$$

Problem 35: Unable to integrate problem.

$$\int \frac{(a+b\,\mathrm{arcsec}(c x))\sqrt{e x^2+d}}{x^6}\,dx$$

Optimal(type 4, 399 leaves, 12 steps):

$$-\frac{(e x^2+d)^{3/2}(a+b\,\mathrm{arcsec}(c x))}{5 d x^5}+\frac{2 e (e x^2+d)^{3/2}(a+b\,\mathrm{arcsec}(c x))}{15 d^2 x^3}+\frac{b c (e x^2+d)^{3/2}\sqrt{c^2 x^2-1}}{25 d x^4\sqrt{c^2 x^2}}$$
$$+\frac{b c (24 c^4 d^2+19 c^2 d e-31 e^2)\sqrt{c^2 x^2-1}\,\sqrt{e x^2+d}}{225 d^2\sqrt{c^2 x^2}}+\frac{b c (12 c^2 d-e)\sqrt{c^2 x^2-1}\,\sqrt{e x^2+d}}{225 d x^2\sqrt{c^2 x^2}}$$
$$-\frac{b c^2 (24 c^4 d^2+19 c^2 d e-31 e^2)\, x\, \mathrm{EllipticE}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right)\sqrt{-c^2 x^2+1}\,\sqrt{e x^2+d}}{225 d^2\sqrt{c^2 x^2}\,\sqrt{c^2 x^2-1}\,\sqrt{1+\frac{e x^2}{d}}}$$

$$+\frac{b\left(c^{2} d+e\right)\left(24 c^{4} d^{2}+7 c^{2} d e-30 e^{2}\right) x \operatorname{EllipticF}\left(c x, \sqrt{-\frac{e}{c^{2} d}}\right) \sqrt{-c^{2} x^{2}+1} \sqrt{1+\frac{e x^{2}}{d}}}{225 d^{2} \sqrt{c^{2} x^{2}} \sqrt{c^{2} x^{2}-1} \sqrt{e x^{2}+d}}$$

Result(type 8, 23 leaves):

$$\int \frac{(a+b \operatorname{arcsec}(c x)) \sqrt{e x^{2}+d}}{x^{6}}\, dx$$

Problem 36: Unable to integrate problem.

$$\int x\left(e x^{2}+d\right)^{3/2}(a+b \operatorname{arcsec}(c x))\, dx$$

Optimal(type 3, 218 leaves, 10 steps):

$$\frac{\left(e x^{2}+d\right)^{5/2}(a+b \operatorname{arcsec}(c x))}{5 e}+\frac{b c d^{5/2} x \arctan\left(\frac{\sqrt{e x^{2}+d}}{\sqrt{d} \sqrt{c^{2} x^{2}-1}}\right)}{5 e \sqrt{c^{2} x^{2}}}-\frac{b\left(15 c^{4} d^{2}+10 c^{2} d e+3 e^{2}\right) x \operatorname{arctanh}\left(\frac{\sqrt{e} \sqrt{c^{2} x^{2}-1}}{c \sqrt{e x^{2}+d}}\right)}{40 c^{4} \sqrt{e} \sqrt{c^{2} x^{2}}}$$
$$-\frac{b x\left(e x^{2}+d\right)^{3/2} \sqrt{c^{2} x^{2}-1}}{20 c \sqrt{c^{2} x^{2}}}-\frac{b\left(7 c^{2} d+3 e\right) x \sqrt{c^{2} x^{2}-1} \sqrt{e x^{2}+d}}{40 c^{3} \sqrt{c^{2} x^{2}}}$$

Result(type 8, 21 leaves):

$$\int x\left(e x^{2}+d\right)^{3/2}(a+b \operatorname{arcsec}(c x))\, dx$$

Problem 39: Unable to integrate problem.

$$\int \frac{x^{5}(a+b \operatorname{arcsec}(c x))}{\sqrt{e x^{2}+d}}\, dx$$

Optimal(type 3, 271 leaves, 11 steps):

$$-\frac{2 d\left(e x^{2}+d\right)^{3/2}(a+b \operatorname{arcsec}(c x))}{3 e^{3}}+\frac{\left(e x^{2}+d\right)^{5/2}(a+b \operatorname{arcsec}(c x))}{5 e^{3}}+\frac{8 b c d^{5/2} x \arctan\left(\frac{\sqrt{e x^{2}+d}}{\sqrt{d} \sqrt{c^{2} x^{2}-1}}\right)}{15 e^{3} \sqrt{c^{2} x^{2}}}$$
$$-\frac{b\left(45 c^{4} d^{2}-10 c^{2} d e+9 e^{2}\right) x \operatorname{arctanh}\left(\frac{\sqrt{e} \sqrt{c^{2} x^{2}-1}}{c \sqrt{e x^{2}+d}}\right)}{120 c^{4} e^{5/2} \sqrt{c^{2} x^{2}}}-\frac{b x\left(e x^{2}+d\right)^{3/2} \sqrt{c^{2} x^{2}-1}}{20 c e^{2} \sqrt{c^{2} x^{2}}}+\frac{d^{2}(a+b \operatorname{arcsec}(c x)) \sqrt{e x^{2}+d}}{e^{3}}$$

$$+\frac{b\left(19 c^2 d-9 e\right) x \sqrt{c^2 x^2-1}\, \sqrt{e x^2+d}}{120 c^3 e^2 \sqrt{c^2 x^2}}$$

Result(type 8, 23 leaves):

$$\int \frac{x^5 \left(a+b \operatorname{arcsec}(c x)\right)}{\sqrt{e x^2+d}} \, dx$$

Problem 40: Unable to integrate problem.

$$\int \frac{x \left(a+b \operatorname{arcsec}(c x)\right)}{\sqrt{e x^2+d}} \, dx$$

Optimal(type 3, 110 leaves, 9 steps):

$$\frac{b c x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\, \sqrt{c^2 x^2-1}}\right) \sqrt{d}}{e \sqrt{c^2 x^2}}-\frac{b x \operatorname{arctanh}\left(\frac{\sqrt{e}\, \sqrt{c^2 x^2-1}}{c \sqrt{e x^2+d}}\right)}{\sqrt{e}\, \sqrt{c^2 x^2}}+\frac{\left(a+b \operatorname{arcsec}(c x)\right) \sqrt{e x^2+d}}{e}$$

Result(type 8, 21 leaves):

$$\int \frac{x \left(a+b \operatorname{arcsec}(c x)\right)}{\sqrt{e x^2+d}} \, dx$$

Problem 41: Unable to integrate problem.

$$\int \frac{x^3 \left(a+b \operatorname{arcsec}(c x)\right)}{\left(e x^2+d\right)^{3/2}} \, dx$$

Optimal(type 3, 133 leaves, 9 steps):

$$-\frac{b x \operatorname{arctanh}\left(\frac{\sqrt{e}\, \sqrt{c^2 x^2-1}}{c \sqrt{e x^2+d}}\right)}{e^{3/2} \sqrt{c^2 x^2}}+\frac{2 b c x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\, \sqrt{c^2 x^2-1}}\right) \sqrt{d}}{e^2 \sqrt{c^2 x^2}}+\frac{d \left(a+b \operatorname{arcsec}(c x)\right)}{e^2 \sqrt{e x^2+d}}+\frac{\left(a+b \operatorname{arcsec}(c x)\right) \sqrt{e x^2+d}}{e^2}$$

Result(type 8, 23 leaves):

$$\int \frac{x^3 \left(a+b \operatorname{arcsec}(c x)\right)}{\left(e x^2+d\right)^{3/2}} \, dx$$

Problem 43: Unable to integrate problem.

$$\int \frac{a+b \operatorname{arcsec}(c x)}{\left(e x^2+d\right)^{3/2}} \, dx$$

Optimal(type 4, 98 leaves, 5 steps):

$$\frac{x\left(a+b\operatorname{arcsec}(c x)\right)}{d\sqrt{e x^2+d}}-\frac{b x \operatorname{EllipticF}\left(c x, \sqrt{-\frac{e}{c^2 d}}\right)\sqrt{-c^2 x^2+1}\,\sqrt{1+\frac{e x^2}{d}}}{d\sqrt{c^2 x^2}\,\sqrt{c^2 x^2-1}\,\sqrt{e x^2+d}}$$

Result(type 8, 20 leaves):

$$\int\frac{a+b\operatorname{arcsec}(c x)}{\left(e x^2+d\right)^{3/2}}\,dx$$

Problem 44: Unable to integrate problem.

$$\int\frac{x^3\left(a+b\operatorname{arcsec}(c x)\right)}{\left(e x^2+d\right)^{5/2}}\,dx$$

Optimal(type 3, 139 leaves, 7 steps):

$$\frac{d\left(a+b\operatorname{arcsec}(c x)\right)}{3 e^2\left(e x^2+d\right)^{3/2}}-\frac{2 b c x \arctan\left(\frac{\sqrt{e x^2+d}}{\sqrt{d}\,\sqrt{c^2 x^2-1}}\right)}{3 e^2\sqrt{d}\,\sqrt{c^2 x^2}}+\frac{-a-b\operatorname{arcsec}(c x)}{e^2\sqrt{e x^2+d}}+\frac{b c x\sqrt{c^2 x^2-1}}{3 e\left(c^2 d+e\right)\sqrt{c^2 x^2}\,\sqrt{e x^2+d}}$$

Result(type 8, 23 leaves):

$$\int\frac{x^3\left(a+b\operatorname{arcsec}(c x)\right)}{\left(e x^2+d\right)^{5/2}}\,dx$$

Test results for the 17 problems in "5.5.2 Inverse secant functions.txt"

Problem 1: Unable to integrate problem.

$$\int\frac{\operatorname{arcsec}(x^5 a)}{x}\,dx$$

Optimal(type 4, 80 leaves, 7 steps):

$$\frac{I\operatorname{arcsec}(x^5 a)^2}{10}-\frac{\operatorname{arcsec}(x^5 a)\ln\left(1+\left(\frac{1}{x^5 a}+I\sqrt{1-\frac{1}{x^{10}a^2}}\right)^2\right)}{5}+\frac{I\operatorname{polylog}\left(2,-\left(\frac{1}{x^5 a}+I\sqrt{1-\frac{1}{x^{10}a^2}}\right)^2\right)}{10}$$

Result(type 8, 12 leaves):

$$\int\frac{\operatorname{arcsec}(x^5 a)}{x}\,dx$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int x^4 \operatorname{arcsec}(b x + a)\, dx$$

Optimal(type 3, 173 leaves, 9 steps):

$$\frac{a^5 \operatorname{arcsec}(b x + a)}{5 b^5} + \frac{x^5 \operatorname{arcsec}(b x + a)}{5} - \frac{(40 a^4 + 40 a^2 + 3) \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{(b x + a)^2}}\right)}{40 b^5} + \frac{a (53 a^2 + 20) (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{30 b^5} + \frac{11 a x^2 (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{60 b^3} - \frac{x^3 (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{20 b^2} - \frac{(58 a^2 + 9) (b x + a)^2 \sqrt{1 - \frac{1}{(b x + a)^2}}}{120 b^5}$$

Result(type 3, 508 leaves):

$$-\frac{((b x + a)^2 - 1) x^3}{20 b^2 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} - \frac{\sqrt{(b x + a)^2 - 1}\, a^5 \arctan\left(\frac{1}{\sqrt{(b x + a)^2 - 1}}\right)}{5 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} - \frac{\sqrt{(b x + a)^2 - 1}\, a^4 \ln\left(b x + a + \sqrt{(b x + a)^2 - 1}\right)}{b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} - \frac{3 ((b x + a)^2 - 1) x}{40 b^4 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} - \frac{\sqrt{(b x + a)^2 - 1}\, a^2 \ln\left(b x + a + \sqrt{(b x + a)^2 - 1}\right)}{b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} - \frac{29 ((b x + a)^2 - 1) x a^2}{60 b^4 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} + \frac{x^5 \operatorname{arcsec}(b x + a)}{5} + \frac{77 ((b x + a)^2 - 1) a^3}{60 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} + \frac{71 ((b x + a)^2 - 1) a}{120 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} - \frac{3 \sqrt{(b x + a)^2 - 1} \ln\left(b x + a + \sqrt{(b x + a)^2 - 1}\right)}{40 b^5 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)} + \frac{11 ((b x + a)^2 - 1) x^2 a}{60 b^3 \sqrt{\frac{(b x + a)^2 - 1}{(b x + a)^2}}\, (b x + a)}$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int x^3 \operatorname{arcsec}(b x + a)\, dx$$

Optimal(type 3, 135 leaves, 8 steps):

$$-\frac{a^4 \operatorname{arcsec}(b x + a)}{4 b^4} + \frac{x^4 \operatorname{arcsec}(b x + a)}{4} + \frac{a (2 a^2 + 1) \operatorname{arctanh}\left(\sqrt{1 - \frac{1}{(b x + a)^2}}\right)}{2 b^4} - \frac{(17 a^2 + 2) (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{12 b^4} - \frac{x^2 (b x + a) \sqrt{1 - \frac{1}{(b x + a)^2}}}{12 b^2} + \frac{a (b x + a)^2 \sqrt{1 - \frac{1}{(b x + a)^2}}}{3 b^4}$$

Result(type 3, 358 leaves):

$$\frac{x^4 \operatorname{arcsec}(b x+a)}{4}-\frac{\left((b x+a)^2-1\right) x^2}{12 b^2 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}+\frac{\sqrt{(b x+a)^2-1}\, a^4 \arctan\left(\frac{1}{\sqrt{(b x+a)^2-1}}\right)}{4 b^4 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}+\frac{\left((b x+a)^2-1\right) x a}{3 b^3 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}$$
$$+\frac{\sqrt{(b x+a)^2-1}\, a^3 \ln\left(b x+a+\sqrt{(b x+a)^2-1}\right)}{b^4 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}-\frac{13\left((b x+a)^2-1\right) a^2}{12 b^4 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}+\frac{\sqrt{(b x+a)^2-1}\, a \ln\left(b x+a+\sqrt{(b x+a)^2-1}\right)}{2 b^4 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}$$
$$-\frac{(b x+a)^2-1}{6 b^4 \sqrt{\frac{(b x+a)^2-1}{(b x+a)^2}}\,(b x+a)}$$

Problem 12: Unable to integrate problem.

$$\int \operatorname{arcsec}(b x+a)^3\, dx$$

Optimal(type 4, 207 leaves, 10 steps):

$$\frac{(b x+a) \operatorname{arcsec}(b x+a)^3}{b}+\frac{6 I \operatorname{arcsec}(b x+a)^2 \arctan\left(\frac{1}{b x+a}+I \sqrt{1-\frac{1}{(b x+a)^2}}\right)}{b}$$
$$-\frac{6 I \operatorname{arcsec}(b x+a) \operatorname{polylog}\left(2,-I\left(\frac{1}{b x+a}+I \sqrt{1-\frac{1}{(b x+a)^2}}\right)\right)}{b}+\frac{6 I \operatorname{arcsec}(b x+a) \operatorname{polylog}\left(2, I\left(\frac{1}{b x+a}+I \sqrt{1-\frac{1}{(b x+a)^2}}\right)\right)}{b}$$
$$+\frac{6 \operatorname{polylog}\left(3,-I\left(\frac{1}{b x+a}+I \sqrt{1-\frac{1}{(b x+a)^2}}\right)\right)}{b}-\frac{6 \operatorname{polylog}\left(3, I\left(\frac{1}{b x+a}+I \sqrt{1-\frac{1}{(b x+a)^2}}\right)\right)}{b}$$

Result(type 8, 10 leaves):

$$\int \operatorname{arcsec}(b x+a)^3\, dx$$

Problem 14: Unable to integrate problem.

$$\int x^{-1+n} \operatorname{arcsec}\left(a+b x^n\right) dx$$

Optimal(type 3, 47 leaves, 6 steps):

$$\frac{(a+b x^n)\operatorname{arcsec}(a+b x^n)}{b n} - \frac{\operatorname{arctanh}\left(\sqrt{1-\frac{1}{(a+b x^n)^2}}\right)}{b n}$$

Result(type 8, 16 leaves):

$$\int x^{-1+n}\operatorname{arcsec}(a+b x^n)\,dx$$

Problem 15: Unable to integrate problem.

$$\int e^{\operatorname{arcsec}(a x)}\,dx$$

Optimal(type 5, 107 leaves, 5 steps):

$$-\frac{(1+I)\,e^{(1+I)\operatorname{arcsec}(a x)}\,\mathrm{hypergeom}\left(\left[1,\frac{1}{2}-\frac{I}{2}\right],\left[\frac{3}{2}-\frac{I}{2}\right],-\left(\frac{1}{a x}+I\sqrt{1-\frac{1}{x^2 a^2}}\right)^2\right)}{a}$$
$$+\frac{(2+2I)\,e^{(1+I)\operatorname{arcsec}(a x)}\,\mathrm{hypergeom}\left(\left[2,\frac{1}{2}-\frac{I}{2}\right],\left[\frac{3}{2}-\frac{I}{2}\right],-\left(\frac{1}{a x}+I\sqrt{1-\frac{1}{x^2 a^2}}\right)^2\right)}{a}$$

Result(type 8, 7 leaves):

$$\int e^{\operatorname{arcsec}(a x)}\,dx$$

Problem 16: Unable to integrate problem.

$$\int \frac{e^{\operatorname{arcsec}(a x)}}{x^2}\,dx$$

Optimal(type 3, 31 leaves, 3 steps):

$$-\frac{e^{\operatorname{arcsec}(a x)}}{2 x}+\frac{a\,e^{\operatorname{arcsec}(a x)}\sqrt{1-\frac{1}{x^2 a^2}}}{2}$$

Result(type 8, 11 leaves):

$$\int \frac{e^{\operatorname{arcsec}(a x)}}{x^2}\,dx$$

Problem 17: Unable to integrate problem.

$$\int \frac{e^{\operatorname{arcsec}(a x)}}{x^4}\,dx$$

Optimal(type 3, 70 leaves, 6 steps):

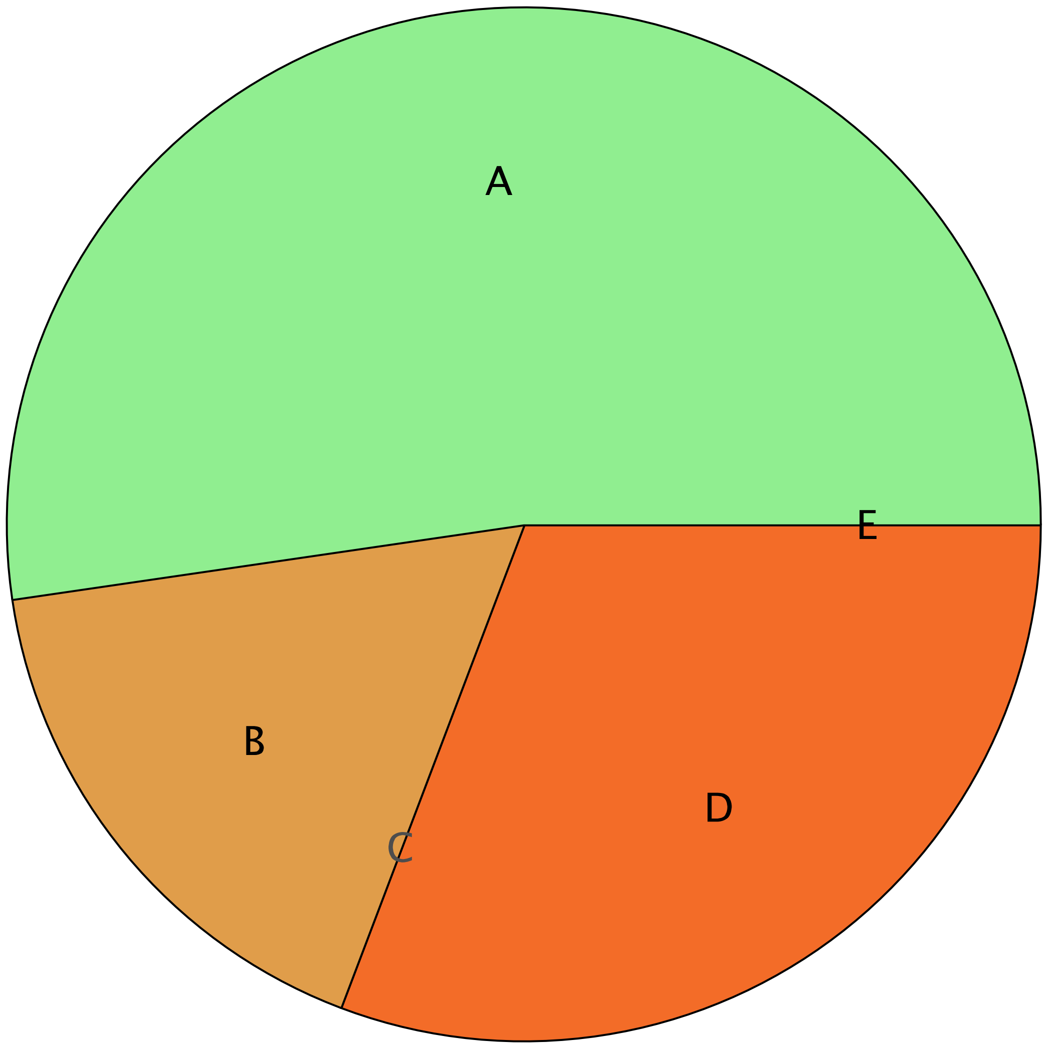
$$-\frac{a^2 e^{\operatorname{arcsec}(a x)}}{8 x}-\frac{3 a^3 e^{\operatorname{arcsec}(a x)} \cos(3 \operatorname{arcsec}(a x))}{40}+\frac{a^3 e^{\operatorname{arcsec}(a x)} \sin(3 \operatorname{arcsec}(a x))}{40}+\frac{a^3 e^{\operatorname{arcsec}(a x)} \sqrt{1-\frac{1}{x^2 a^2}}}{8}$$

Result(type 8, 11 leaves):

$$\int \frac{e^{\operatorname{arcsec}(a x)}}{x^4}\, dx$$

Summary of Integration Test Results

65 integration problems



A - 34 optimal antiderivatives
B - 11 more than twice size of optimal antiderivatives
C - 0 unnecessarily complex antiderivatives
D - 20 unable to integrate problems
E - 0 integration timeouts